Math Problems for Pilots, by Merlin Woodman

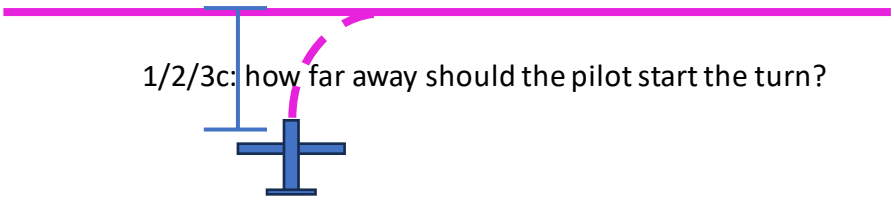
Joining a Course

Context

Instrument pilots often fly a series of different headings to avoid weather or traffic before intercepting a course and continuing in that new direction. In the example above, you see the navigation display of a Cessna 172 where the airplane is approaching the pink course line at close to a right angle. The pilot will want to make a right turn onto that course to proceed to the highlighted point, CEBOD. At the top of the nav display in the red highlight box, we can see the ground speed (GS) of the airplane in knots (nautical miles per hour), and the cross-track error (XTK), or the distance from the course. To join that new course, they need to lead out their turn. If the pilot turns early or late, they will need to spend more time correcting, so we want to come up with an answer that's precise enough to help them get it right the first time.

Given Information and Assumptions

- The airplane is moving at a ground speed of 86 knots (nm/hr).
- The airplane should turn at a rate of 3 degrees per second.
- The airplane is exactly 90 degrees off its desired course.
- There is no wind drift.



Questions to Answer

1. For 86 knots:
    a. How long will it take the airplane to complete the 90-degree turn?
    b. How much distance will the airplane cover during that turn?
    c. How far away from the course must the pilot turn in order to intercept it? In other words, we need to tell the pilot "Start your turn at ___ XTK."
2. Let's change the scenario a bit: the airplane is now moving at 100 knots.
    a. How long will it take the airplane to complete the 90-degree turn?
    b. How much distance will the airplane cover during that turn?
    c. How far away from the course must the pilot turn in order to intercept it? In other words, we need to tell the pilot "Start your turn at ___ XTK."
3. One final scenario: the airplane is now moving at 120 knots.
    a. How long will it take the airplane to complete the 90-degree turn?
    b. How much distance will the airplane cover during that turn?
    c. How far away from the course must the pilot turn in order to intercept it? In other words, we need to tell the pilot "Start your turn at ___ XTK."
4. What patterns do we start to see when we compare the 86, 100, and 120 GS scenarios?
    a. For the a) questions, what pattern is there?
    b. What relationship between speed and distance do we see in b?
    c. What relationship between speed and lead-out distance do we see in c?
5. After seeing what happens with 86, 100, and 120 knot GS, we now want to make a generic rule for pilots to follow: "To intercept a course at any given GS, start your turn at [some function] of that GS." What's a rule you can come up with based on 4c that you think pilots should follow? Test it with a few sample speeds (e.g. GS=70, GS=130, etc.) to see if it works.
6. Discuss your answers to Q5 with some classmates.
    a. Which rules do you think provide the best precision?
    b. Which ones are simplest in terms of mental math?
    c. If your nav display only gave you .1 nm increments, which rule would you select?

d. If, like the Cessna 172 pilot approaching the course above, you were in the airplane and had to come up with an answer in 10 seconds, which rule would you use?

Conclusions

Hopefully you and your classmates came up with a couple of slightly different answers to Q5 and discussed not only the techniques, but also tradeoffs, associated with each one. You may have also had diverging answers to Q6, which is great. Some people are good at doing complex calculations quickly. Others prefer something simpler and more foolproof. If the *goal* of the exercise is to help pilots get things pretty close, then there can be several correct answers that are all fit for purpose.